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Fermions Between One and Two Dimensions

T. M. Rice [a]

[a] Theoretische Physik, ETH Hönggerberg, CH-8093, Zürich, Switzerland

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Fermions Between One and Two Dimensions

T. M. RICE

Theoretische Physik, ETH Hönggerberg, CH-8093 Zürich, Switzerland

When fermions are confined to move only in one dimension, as in a chain, then they form a Luttinger Liquid which differs fundamentally from the Landau Fermi Liquid that occurs in higher dimensions. If chains are combined to form ladders, one finds surprisingly that the properties of fermionic ladders depend essentially on their width. Ladder can be divided in two classes, those with odd and even numbers of legs. To date this behavior has been observed only in certain cuprate materials and the ideal of lightly doped and perfectly formed ladders has not been realized. The synthesis of materials with ladder structures should lead to interesting new physics.

Keywords: quantum magnetism; spin ladders

I. Introduction

The properties of electrons confined to move along a chain have been studied for many years[1]. Some of the best examples are to be found among the organic conductors[2]. Electrons confined to move in a plane have also been studied very intensively since the discovery of the phenomenon of high-Tc superconductivity in the planar cuprates[3]. One might expect that a smooth crossover from chains to a plane would result if one assembles chains to form ladders of increasing width. Recently it was found[4] that this expectation is totally false. Instead the properties are very depen-

dent on the width or number of legs in the ladder – a surprising discovery. In this lecture the properties of such ladder systems will be reviewed.

First the case of two leg ladder with exactly one electron per site will be discussed. In this stoichiometric limit the problem can be viewed most simply in the limit of strong Coulomb interactions which reduces to the study of an antiferromagnetic (AF) S=1/2 Heisenberg model. The ground state is a spin liquid – so called because the spin-spin correlation functions are strictly short range with a finite correlation length, ξ. The term spin liquid is really a misnomer because the groundstate is actually a quantum coherent singlet state with a finite energy gap to the lowest excited triplet, usually designated as the spin gap. This quantum behavior runs counter to one's intuition based on classical physics that coupling two chains in this unfrustrated manner should strengthen the power law decaying AF correlations rather than replace them with an exponential decay. The two leg ladder can also be considered in the weak coupling limit, where a qualitatively similar behavior is found, i.e. a groundstate with a gap for charge excitations and short range spin-spin correlations. Balents and Fisher[5] introduced a notation which classifies a groundstate according to the number of gapless charge and spin modes, in this case C0S0. In this case there is continuity between weak and strong coupling. Adding another leg to form a 3-leg ladder changes qualitatively the nature of the groundstate. The low energy behavior in the strong coupling limit is now that of a single chain i.e. gapless excitations and a power law decay of the spin-spin correlations. Similarly in the weak coupling limit the groundstate is classified as C0S1. This difference persists between the groundstates of ladders with an even and an odd number of legs. The former have a finite spin gap which however drops rapidly with increasing width, while the latter have a single gapless mode.

This intriguing behavior continues when one examines the effect of hole doping. The single chain forms of course the well known Tomonaga-Luttinger liquid[1]. The 2-leg ladder is quite different when doped and instead forms as a Luther-Emery liquid. The spin gap persists and the doped holes pair up in an approximate d-wave state. These hole pairs behave as a liquid of bosons and show power law correlations in the charge density and bose condensation correlation functions. Which one dominates and has the slower decay depends on the details of the interaction between hole pairs. Again there is a fundamental difference between even and odd leg ladders. In the latter case the holes first enter the gapless transverse channel and at least for an initial hole concentration form a Tomonaga-Luttinger liquid. This TL-liquid is stable up to a finite critical concentration of holes due to the finite energy difference to introduce holes between the gapless channels and those with a spin gap. This then leads to a highly unusual situation because the Fermi surface with a number of channels (or bands) equal to the width is truncated down to a single channel although there is no superlattice, neither spin nor charge. Unfortunately there is no experimental realization of such a system at present.

A number of cuprate compounds with ladder structures have been synthesized to date. The predicted behavior of the undoped ladders has been verified but for the effects of hole doping the only cuprate system at present $Sr_{14-x}Ca_xCu_{24}O_{41}$, is not the ideal system to test the theoretical ideas.

In this lecture the properties of undoped ladder systems is reviewed first in chapter 2. Then the effect of doping is described in chapter 3 and this brief review concludes with a summary of the experiments on cuprates which have ladder structures and of the challenge the experimental realization of such ladder systems in organic compounds poses.

II. Undoped Ladder Systems

In the limit of strong onsite repulsion a single band at half-filling has only spin excitations at low energy defined by a S=1/2 Heisenberg AF model. The properties of this model on 1D chains or on 2D square lattices are well-known. The model is defined by the Hamiltonian

$$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j \tag{1}$$

where i is a vector labeling the lattice sites on which spin-1/2 operators $\vec{S}_i$ are located, $\langle i, j \rangle$ denotes nearest-neighbor sites, and $J(> 0)$ is the AF exchange coupling that provides the energy scale in the problem. On 2D square lattices, the Heisenberg model has a ground state with long-range AF order[6]. Whereas in 1D chains, quantum effects prevent long-range AF order and the spin–spin correlation decays slowly to zero as a power law in the site separation. Neither system has a spin gap, that is, there is no cost in energy to create an excitation with $S = 1$.

The field of ladder systems started when Dagotto et al.[7] (see also[8,9]) found evidence that 2-leg ladders have a finite spin gap, in other words a finite energy is needed to create a $S = 1$ excitation. They started with the simple limit obtained by generalizing Eq. 1 so that the exchange coupling along the rungs of a 2-leg ladder (denoted by $J_\perp$) is much larger than the coupling J along the chains. The energy in this limit is approximately $E_{gs} = -3/4 J_\perp N$, where N is the number of rungs and $-3/4 J_\perp$ is the energy of each rung singlet state. The ground state has a total spin $S = 0$ and to produce a $S = 1$ excitation, a rung singlet must be promoted to a $S = 1$ triplet. The coupling along the chains creates a band of $S = 1$ magnons with a dispersion law

$$\omega(k) = J_\perp + J \cos k \tag{2}$$

in the limit $J_\perp \gg J$ (k is the wave vector). The spin gap is the minimum excitation energy $\triangle_{\text{spin}} = \omega(\pi) \approx J_\perp - J$, which remains large in this limit. Concurrently, the spins are mostly uncorrelated between rungs so that the spin correlations decay exponentially with distance along the chains leading to the name spin-liquid for this type of state. Note, however, that the spins are not disordered but rather are in an isolated quantum-coherent ground state.

In the other extreme, $J_\perp/J = 0$, the two chains decouple, but isolated spin-1/2 Heisenberg chains do not have a spin gap and excitations with $S = 1$ and $k = \pi$ are degenerate with the ground state. Barnes et al.[10] observed that the power-law decay of the spin correlation in an isolated chain implies that a chain is in a critical state, and thus small perturbations can qualitatively alter its properties. They predicted that the spin gap $\triangle_{\text{spin}} > 0$ for all $J_\perp/J > 0$, including the values of experimental interest, $J_\perp/J \sim 1$.

The ladder spin system would always be in a spin-liquid state, in contrast to the more familiar cases of the 1D and 2D Heisenberg models, which are gapless. Numerical techniques, exact diagonalization of small clusters[10] and quantum Monte Carlo techniques, have been used to study $\triangle_{\text{spin}}$ as a function of $J_\perp/J$. Recently using a powerful new algorithm, Greven et al.[11] reported quantum Monte-Carlo simulations on systems with a wide range of values $J_\perp/J$. They found values $\triangle_{\text{spin}}(J_\perp/J) \approx 0.41 J_\perp$ as $J_\perp/J \to 0$ in agreement with the results of field theoretic analyses[12]. The results are shown in Figure 1.

At the isotropic coupling $J_\perp = J$, White et al.[14], using a density matrix renormalization group (DMRG) technique suitable for static properties of 1D systems, reported $\triangle_{\text{spin}} = 0.504J$, in excellent agreement with the QMC results[11]. There are AF spin correlations at short distances

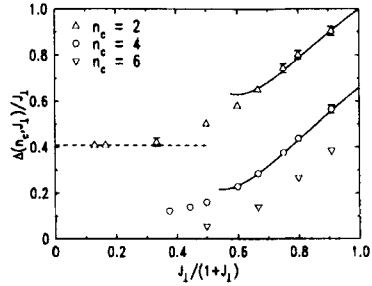

Figure 1: The dependence of the spin gap, $\triangle_{\text{spin}}$ in $S = 1/2$ AF Heisenberg spin ladders on the rung coupling, $J_\perp$ and number of legs, n_c as calculated by Greven et al.[11] from QMC simulations on finite but large systems. (The coupling along the legs, $J = 1$.) The solid and dashed lines show the behavior predicted by strong and weak coupling expansions[13,12].

along the chains and across the rungs, but even at $J_\perp = J$, the latter are somewhat stronger[14], showing that the rough picture of a ground state dominated by rung singlets is robust. Gopalan et al.[15] suggested that a good variational description of the ground state could be obtained using the short-range resonance valence bond state proposed by Anderson[16] and Kivelson et al.[17] with mostly adjacent rung singlets but including resonance between two adjacent rung singlets into two nearest-neighbor singlets along the chains.

The limit $J_\perp/J \gg 1$ again allows us to see what happens when we increase the number of chains, n_c keeping n_c even. Clearly there is a finite spin gap in this limit but $\triangle_c(n_c)$ will decrease markedly as n_c increases. Poilblanc et al.[18] studied finite size clusters with 4 legs and extrapolating to an infinite ladder found a value $\triangle_{\rm spin}(n_c = 4,\ J_\perp = J) = 0.245J$ — a reduction of 50% with respect to $n_c = 2$. White et al.[14] used a DMRG technique on larger clusters and obtained a smaller value, $0.19J$. The most reliable value is that of Greven et al.[11] from QMC who report a value $\triangle_{\rm spin}(n_c = 4,\ J_\perp = J) = 0.160\ J$. At $n_c = 6$, the decrease they find is even more marked $\triangle_{\rm spin}(n_c = 6,\ J_\perp = J) = 0.055J$. These results indicate a finite spin gap $\triangle_{\rm spin}$ for even values of n_c, but an exponential decay of $\triangle_{\rm spin}$ with increasing n_c. Greven et al.[11] also report extensive results on the temperature (T) dependence and width dependence of the AF correlation length $\xi(n_c, T)$ which controls the asymptotic exponential decay of staggered spin–spin correlation function

$$\begin{aligned} C(i,j) &= (-1)^{\mathrm{sign}(i,j)} \langle \vec{S}_i \cdot \vec{S}_j \rangle \\ &\sim r_{ij}^{-1/2}\, e^{-r_{ij}/\xi} \quad \text{as} \quad r \to \infty \end{aligned} \tag{3}$$

where sign $(i,j) = \pm 1$ if i and j are separated by even (or odd) number of bonds. The values of $\xi(n_c, T \to 0)$ at isotropic couplings ($J_\perp = J$) increases markedly with n_c from $\xi(n_c = 2) = 3.24$ to $\xi(n_c = 4) = 10.3$ to a value ~ 30 at $n_c = 6$[11].

The single magnon spectrum, $\omega(k)$, evolves from the simple cosine form, Eq. (2) at $J_\perp \gg J$ to a nearly linear form near the minimum at $k = \pi$ when $J_\perp = J$. At low temperatures, thermally excited magnons act as a low density gas of $S = 1$ spins and lead to an exponentially small value of the spin susceptibility $\chi(T)$ at $T \ll \triangle_{\rm spin}$

$$\chi(n_c, J_\perp, T) \sim T^{-1/2}\, e^{-\triangle_{\rm spin}(n_c, J_\perp)/T}\ . \tag{4}$$

Early calculations by Troyer et al.[19] used a quantum transfer method and more recent calculations by Frischmuth et al.[20] use the powerful loop algorithm in QMC to reach values $T \ll J$ in systems with a large number of spins. Figure 2 displays their results showing the crossover from an exponential behavior at $T \ll J$ to a Curie-Weiss form for $T \gtrsim J$. The strong dependence of Δ_{spin} on n_c is very clear in this work also.

The results in Figure 2, show that the ladders with n_c odd behave in a very different way as $T \to 0$. This is in line with conjectures by Hirsch and Tsunetsugu (see Reference [21]) that odd-leg ladders should behave quite differently from even-leg ladders and display properties similar to single chains at low energies, namely gapless spin excitations and a power-law falloff of the spin–spin correlations.

The simplest way to visualize this difference is again by analyzing the large $J_\perp/J$ limit where each rung can be diagonalized exactly, leading to a doublet ground state. The interrung coupling J generates an effective interaction between these doublet S=1/2 rung states. By rotational invariance this interaction must be of the Heisenberg form with an effective coupling, J_{eff}. Thus, the ground state properties of the 3-leg ladder at large $J_\perp/J$ should be those of the spin-1/2 Heisenberg chain with a coupling J_{eff} instead of J, leading to a vanishing spin gap. The argument can be trivially generalized to all odd-leg ladders. Because there is no spin gap for the odd-leg case at both $J_\perp/J \gg 1$ and $J_\perp/J = 0$, it is reasonable that the gap vanishes at intermediate values of $J_\perp/J$, in contrast to even-leg ladders.

Khveshchenko[22] explained the qualitative difference between even- and odd-leg ladders on the basis of an argument used by Haldane for the 2D square lattice[23]. For odd-leg ladders, a topological term governing the dynamics at long wavelengths appears in the effective action, whereas for

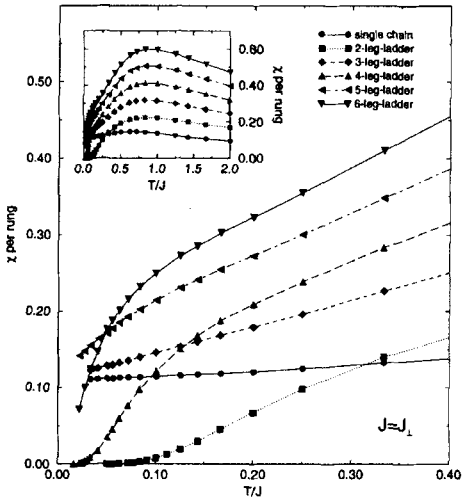

Figure 2: The temperature dependence of the uniform spin susceptibility $\chi(T)$ of spin ladders calculated by Frischmuth et al.[20] using QMC simulations on finite but large systems. The coupling is isotropic ($J_\perp = J$) and $\chi(T)$ is normalized per rung of n_c spins where n_c is the number of legs in the ladder. Noteworthy as discussed in the text, is the common extrapolation to $T = 0$ K for ladders with $n_c = 1$, 3 and 5.

even-leg ladders, it exactly cancels. This topological term is similar to the one that causes the well-known difference between integer Heisenberg spin chains, which have a finite spin gap, and half-integer spin chains, which are gapless.

The detailed form of the effective $S = 1/2$ Heisenberg model that governs the low T behavior for values of $n_c = 3$ is of interest. A DMRG study by White et al.[14] and the QMC simulations by Frischmuth et al.[20] lead to the surprising conclusion that the spinon velocity, v, of the effective model is essentially unrenormalized and independent of n_c, the width of

the ladder. This is illustrated in Figure 2 which shows that for isotropic coupling the $\lim_{T\to o} \chi(n_c, T)$ per rung of n_c spins, is independent of n_c. At low T the susceptibility of single Heisenberg chain has the form[24]

$$\chi(T) = \frac{1}{4\pi v}\left[2 + (ln\,(T_0/T))^{-1} - ln\,(ln\,(T_0/T) + 1/2)\ /\ 2ln^2\,(T_0/T)\right] . \tag{5}$$

It is evident from Figure 2 that altho' the spinon velocity, v, does not change appreciably with n_c, the temperature scale, T_0, which is determined by spinon-spinon interactions is reduced substantially as n_c increases. Frischmuth et al.[25] examined the cause of this change and showed that it is due to longer range exchange couplings in the effective model. The key point is that these longer range couplings are unfrustrated, i.e., the n.n.n. coupling is ferromagnetic and the nnnn antiferromagnetic. Thus altho' the low energy model is a single $S = 1/2$ model per rung, the interactions in the effective model are considerably renormalized and strongly dependent on the width, n_c. The weak coupling limit of a single band Hubbard model for ladders gives qualitatively similar results[5,26] . At half-filling Umklapp processes scale to strong coupling and introduce a charge gap, i.e. a finite gap for any charge excitations. The spin properties however depend on the width, n_c with a finite spin gap for even values of n_c but gapless excitations for odd values of n_c. In the Balents-Fisher notation these groundstates are classified as C0S0 and C0S1 respectively. The gaps are exponentially small in the weak coupling limit but there is a smooth crossover between weak and strong coupling. Thus these are robust properties which do not depend on the strength of the interactions.

III. Doped Ladders

We consider next the effects of doping with a small density of holes. The simplest model that includes the essentials of the low energy physics is the

t-J model[16,27]. The presence of a projection operator $P_0 \left(= \prod_i (1 - n_{i\uparrow} n_{i\downarrow}) \right)$ ensures that only configurations with an empty and a singly occupied orbital are allowed on each site. The first term represents the hopping process which interchanges the two configurations.

$$H = P_0 \left\{ -t \sum_{\langle ij \rangle} c^+_{i\sigma} c_{j\sigma} + h.c. \right\} P_0 + J \sum_{\langle ij \rangle} \left(\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} n_i n_j \right) . \quad (6)$$

The phase diagram of the t-J model for a single chain was examined some years ago by Ogata and coworkers[28]. They found that in the parameter range of interest for the cuprates, $J/t \sim 1/3$ and hole doping, $\delta (= 1 - n) \sim 0.2$, a Luttinger liquid state is stable with an exponent $K_\rho \sim 0.6$. In this exponent range ($K_\rho < 1$), the predominant correlations are SDW (spin density ware) with an incommensurate period $2k_F (= \pi n)$. The power law commensurate AF correlations at $\delta = 0$, evolve into similar correlations with a larger exponent and incommensurate periodicity upon doping and pairing correlations are suppressed.

The 2-leg ladder, as discussed above, starts from a very different stoichiometric insulating state so one can expect a different behavior when doped. Again if one starts from the limit, $J_\perp \gg J$ the behavior is easy to understand. In this limit, holes pair on the same rung to minimize the loss of magnetic energy. The spin gap remains finite although now there are two quite separate triplet excitations[29,30]. One is the magnon similar to the undoped system which is made by exciting a rung singlet to a triplet. However, the presence of hole pairs introduces a new type of triplet excitation which is created by placing the holes on separated rungs with a triplet configuration of the two free spins. This new excitation has an energy gap unrelated to $\triangle_{\text{spin}}$ of the original magnon. This illustrates the point that hole doping is a singular perturbation of the stoichiometric insulator.

A variety of numerical studies show that the hole pairs and the spin

gap persists down to the isotropic limit $J_\perp = J$ [7,29–31]. Further, the relative state of hole pairs is an approximate 'd-wave' state with opposite signs for the pairing amplitude along the rungs and legs in agreement with the predictions of mean field theory[32]. Also the calculations on finite clusters showed that the low energy excitations are only in the charge sector and correspond to a sound mode of the hole pairs. These results show that the state of a lightly doped ladder is not a Luttinger liquid but instead corresponds to a Luther–Emery liquid[1,33]. As Efetov and Larkin[34] showed, the low energy sector of such a liquid is represented as a fluid of bosons — in this case the Efetov–Larkin bosons are hole pairs primarily confined to a single rung and n.n. leg configurations. The properties of the Bose fluid depend on the effective interaction between the bosons. Estimates for the t-J model[35] and the closely related Hubbard model[36] suggest that the CDW correlations corresponding to crystalline correlations of the bosons are longer range than the superconducting (or bose condensate) correlations.

The lightly doped 3-leg ladder is also of interest. In this case the 3 bands can be split according to their symmetry under reflection about the center leg into even (2-bands) and odd (1-band). The numerical results of exact diagonalization[37] and DMRG[38] calculations show a finite difference in energy to remove an electron (i.e. to add a hole) from the even and odd bands. Remember as discussed above, the undoped 3-leg ladder reduces to an effective $S = 1/2$ single chain and taking the electron out of the odd channel corresponds to this transverse channel. As a result a Tomonaga-Luttinger liquid is formed in the odd channel. Meanwhile the even parity channels remain gaped and insulating until a finite critical hole concentration is reached, at which point holes enter the even parity channels in pairs. This regime in which the band theory Fermi surface with 6

Fermi points is truncated into 2 Fermi points is specially interesting since this Fermi surface truncation is not due to any superlattice or breaking of translation symmetry along the legs of the ladder. Instead it is a consequence of the formation of an insulating spin liquid (ISL) in the even channels. The key to this development is that the Umklapp scattering in the even channels scales to strong coupling and opens both charge and spin gaps in these channels. At present efforts are underway to extend these ideas to planes and a 2-dimensional Fermi surface[39].

The weak coupling limit yields closely related but not identical results. The 2-leg ladder when doped also shows Luther-Emery behavior when doped which in the Balents-Fisher classification appears as a C1SO phase, i.e. only gapless charge modes[26]. The 3-leg ladder is rather different since now one introduces the interactions about the band theory Fermi surface which has 6 Fermi points. The result is a coexisting Tomonaga-Luttinger liquid in the odd channel and a Luther-Emery liquid in the even channels appears at arbitrarily small hole doping. The effect of the finite charge gap in the even channels does not show up in the weak coupling approach[26]. Apart from this difference the weak and strong coupling approaches are quite similar.

IV. Cuprate Ladder Materials

At present the best examples of ladders are to be found among the cuprate compounds. The cuprates are special among the 3d-metal oxides in that they can be doped to give highly mobile carriers when holes in the form of low spin Cu^{3+}-ions are introduced. So there is a great potential here to synthesize materials which can test the theoretical ideas discussed above. This approach, combining as it does reliable calculations of strongly correlated models and experiments on suitably arranged

cuprates, is a promising research direction and should give us not only new perspectives on the whole field of cuprates, but also new insights into the subtle interplay between magnetism and superconductivity that lies at the heart of this field. At present there are three families of cuprate materials which have ladders as their essential structural components.

The first one is the family with chemical composition $Sr_{2n-2}Cu_{2n+2}O_{4n}$, (n: odd) discovered by Hiroi, Takano and coworkers[40,41]. These materials have a simple planar structure with alternating Sr and $Cu_{n+1}O_{2n}$-planes up the c-axis. The key feature of such $Cu_{n+1}O_{2n}$-planes was pointed out a few years ago by Rice et al.[21]. They consist of ladders with each Cu-ion connected by the strong 180° Cu-O-Cu bonds and the interladder bonds arise through edge sharing CuO_4 squares and so are weaker 90° Cu-O-Cu bonds. Additionally the interladder interactions are magnetically frustrated through the triangular coordination that connects the ladders. The structure is illustrated in Figure 3. In the undoped form, the ladders have strong essentially isotopic AF coupling but frustrated weaker F coupling between ladders. Therefore one can expect the interladder coupling to be weak — a result found in a mean field treatment by Gopalan et al.[15]. The magnetic results of Azuma et al.[42] on the 2-leg and 3-leg ladder compounds $SrCu_2O_3$ and $Sr_2Cu_3O_5$ ($n = 5$) agree well with the properties expected for ladders with $n_c = 2$ and 3 respectively. The results show an exponential fall off in $\chi(T)$ as $T \to 0$ for $SrCu_2O_3$ while $\chi(T) \to$ constant as $T \to 0$ in $Sr_2Cu_3O_5$ — the latter even shows AF ordering below a $T_N \simeq 50$ K presumably due to the coupling up the c-axis[43]. Unfortunately, it has not proved possible to dope these compounds, may be because of their close similarity to the infinite layer cuprate, $SrCuO_2$, which is also difficult to dope.

A second family, investigated by Hiroi and Takano[44], is the high pres-

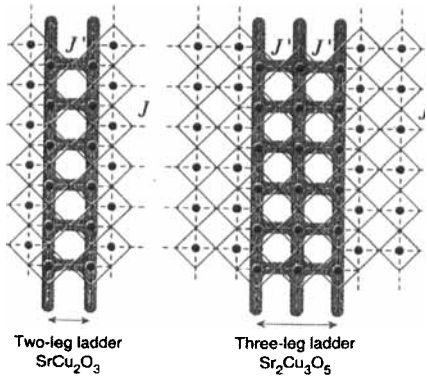

Figure 3: The copper oxide planes in the family $Sr_{2n-2}Cu_{2n+2}O_{4n}$ [41-2]. Shown are the two- and three-leg ladders of strong 180° Cu-O-Cu bonds in the compounds $SrCu_2O_3$ and $Sr_2Cu_3O_5$ (left and right). The solid dots denote the Cu^{2+}-ions and the O^{2-}-ions sit at the intersections of the solid lines.

sure phase of $La_{2-x}Sr_xCuO_{2.5}$. The structure of the undoped $x = 0$, parent compound is shown in Figure 4. The 2-leg ladder structures are easily identified. However in this structure the point symmetry of the Cu- and O-ions is low, leading to substantial interladder coupling as estimated by Normand and Rice[45]. Further the overall structure of the Cu-ions is unfrustrated. The early experiments by Hiroi and Takano[44], found a decrease in $\chi(T)$ at $T \lesssim 300$ K which they interpreted as evidence of a spin gap but then subsequent experiments by Matsumoto et al.[46] found evidence of AF-order below a $T_N = 117$ K. This led Normand and Rice[45] to propose that $LaCuO_{2.5}$ was near to the quantum critical point that separates the spin liquid AF ordered phases and just on the AF-ordered side. Subsequent numerical calculations by Troyer et al.[47] showed that such an

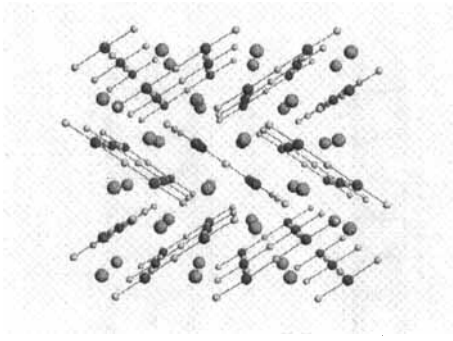

Figure 4: Crystal structure of the depleted perovskite ladder structure in the high pressure phase of $LaCuO_{2.5}$ [44]. The larger spheres denote La^{3+}-ions, the medium size Cu^{2+}-ions and the smallest spheres O^{2-}-ions. The interladder coupling is through O-ions which are members of CuO_4 squares in one ladder and are apical to a CuO_4-square in a neighboring ladder.

occurrence could account for the form of $\chi(T)$. Hiroi and Takano[44] found it possible to substitute Sr^{2+} for the La^{3+}-ions, and so dope the ladders. Unfortunately the materials were only good conductors for relatively large values of x ($x \gtrsim 0.2$) but not superconductors, may be because at these x-values, they are overdoped. At smaller x-values it seems the disorder potential of the La^{3+}/Sr^{2+}-ions is too strong and conductivity is poor also without superconductivity. There are also other related ladders containing La-cuprates with more complex structures[48].

The last family has the chemical formula $A_{14}Cu_{24}O_{41}$. These materials were synthesized and studied already in the late '80's[49]. Further in some cases single crystals were obtained. However, recently they have been reexamined[50]. Altho' the chemical formula is rather complex, the structural form is simpler. There are two components in the form of cuprate

plane which stack alternately. One component has the Cu_2O_3 planes we encountered earlier (Figure 3) and so has weakly coupled 2-leg ladders. The second component has planes with sets of CuO_2-chains widely separated. These CuO_2-chains are made from edge sharing CuO_4-squares and so have only the weak 90° Cu-O-Cu bonds. The A_{14}-ions sit in between the cuprate planes as illustrated in Figure 5. The result is a weakly coupled set of Cu_2O_3-planes with 2-leg ladders. The formula $Cu_{24}O_{41}$ is interpreted as $7 \times Cu_2O_3 + 10 \times CuO_2$ with ratio 7:10 determined by the relative Cu-Cu separations, along the chain/ladder direction.

The next question is the valence state of the Cu-ions in the CuO_2-chains and of course the Cu_2O_3-ladders. This question was examined by Carter et al.[50]. Starting from the compounds with $A_{14} = La_6^{3+}Ca_8^{2+}$, which will be the stoichiometric insulator, i.e. all Cu^{2+}, if the O-concentration is the ideal O_{41}, Carter et al. interpreted the reduction of $\chi(T)$ with substitution of 2+ ions for the 3+ La-ions, as evidence that the holes introduced on Cu-sites formed Cu^{3+} singlets[27] on the CuO_2-chains. The Cu_2O_3-ladders are magnetically inert for $T \lesssim 300$ K due to the spin gap. In their view the compound with only Sr (i.e. $A_{14} = Sr_{14}$) had stoichiometric Cu_2O_3 ladders (all Cu^{2+}) and the 10 Cu sites on the CuO_2 chains were assigned valences, $6 \times Cu^{3+}$, and $4 \times Cu^{2+}$. Thus all the holes would be on the chains and the poor conductivity was interpreted as due to localization in the 1D chains. Recent magnetic studies[51] confirm this assignement.

Ca substituted samples became more conducting and Uehara et al.[52] reported superconductivity with a $T_c \lesssim 10$ K in a narrow pressure range around $P \sim 3$ Gpa. Considerable light was shed on the effect of Ca substitution on the hole distribution between chains and ladders by the work of Osafune et al.[53] who synthesized large single crystals with varying Ca content. This allowed them to make d.c. and optical conductivity mea-

surements polarized parallel and perpendicular to the common axis of the chains and ladders. Their results led them to conclude that even in the pure Sr compound, 1 of the 6 holes per f.u. actually is on the ladders and that Ca substitution increases the hole concentration on the ladders. The d.c. conductivity results of the same group[54] also are very interesting. They show a surprisingly large anisotropy ratio ($\gtrsim 20$) between parallel and perpendicular and further a crossover from conducting to insulating behavior along the ladders as $T \lesssim 50$ K. The superconductivity found by Uehara et al.[52] appears then when this insulating behavior is suppressed by applying pressure.

Detailed structural studies have recently been made only at room temperature and pressure on the Ca-rich compound $Ca_{13.6}Sr_{0.4}Cu_{24}O_{41}$ by Ohta et al[55]. These show that there is a strong interplay between the CuO_2-chains and the ladders in Cu_2O_3-planes. This takes the form of a twisting of the CuO_2-chains so that some O-ions act as apical O-ions for the CuO_4 squares in the ladders.

The structure shows that these apical O-ions tend to appear on around 50% of the rungs and simultaneously on both Cu-ions on a rung. These apical negatively charged O-ions will give rise to attractive potentials for holes on the ladders. The result is that there are substantial potential variations along the ladders for the holes and this makes this system less than ideal to study the physics of lightly doped ladders.

V. Conclusions

In this lecture the interesting physics of ladder systems of fermions was briefly reviewed. In the ladders the fermions are between one and two dimensions but remarkably their behavior differs markedly from either of the limiting cases of one and two dimensions. Clearly for this field to develop;

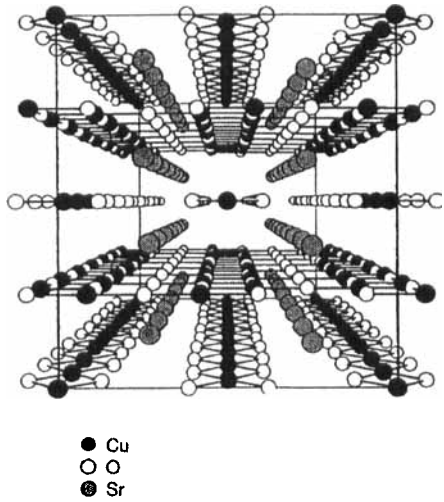

Figure 5: Crystal structure of the family $A_{14}Cu_{24}O_{41}$ [49]. There are alternating cuprate and A-ion layers and the cuprate layers in turn have CuO_2-chains and Cu_2O_3-ladders as their key structural units. The solid dots denote Cu, the open circles O and the large dots A (= Sr, Ca or La) ions.

suitable experimental systems are of paramount importance. At present one has only a limited number of cuprate compounds. Organic compounds have potential here and the synthesis of ladder systems, especially doped ladder systems, is a clear challenge.

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